MATH RELAYS 2023
PROBABILITY & STATISTICS

Place your answer on the appropriate blank of the answer sheet. Express each answer accurate to 3 decimal places.

For problems 1-7 consider the sample data set: 9, 2, 3, 3, 3, 1, 7

1. Find the median.

2. Find the range.

3. Find the mode.

4. Find the sample mean.

5. Find the sample variance.

6. Find the sample standard deviation.

7. Find the standard error of the mean.

For problems 8-9, consider the following discrete distribution.

X	0	1	2	3	4
$P(X)$	0.1	0.2	0.4	0.2	0.1

8. Find the mean of the distribution.

9. Find the variance of the distribution.

10. If the mean of X is 10, then find the mean of $Y = 3X - 2$.

11. If the variance of X is 4, then find the variance of $Y = -2X + 3$

For problems 12 and 13, use the following information.

x	1	2	3	4	5
y	3.1	3.5	3.8	4.6	4.2

12. Find the slope of the least square regression line.

13. Find the y-intercept of the least square regression line.

14. What is the predicted value of y when $x = 2$?

15. Consider two events, A and B such that $P(B) = 0.35$ and $P(A \cup B) = 0.60$. Find $P(A' \cup B)$.

16. Consider two events, A and B such that $P(A \cap B) = 0.15$, and $P(A \cup B') = 0.55$. Find $P(A \mid B)$.

17. Let A and B are two independent events such that $P(A) = 0.4$ and $P(A' \cap B') = 0.42$. Find $P(B)$.

18. If somebody invests \$10,000 at 3.1%, \$6,000 at 2.5%, and \$4,000 at 2%, then find the overall percentage yield. Give the answer as a percentage.

19. Among the 10 candidates for five positions on a city council, 5 are Democrats and 5 are Republicans. In how many different ways can the 5 councilmen be chosen randomly so that 2 are Democrats and 3 are Republicans?

20. In a lot of 10 light bulbs, there are 3 defective bulbs. An inspector selects 2 bulbs one at a time without replacement and tests them. What is the probability that at most one bulb is defective between the two?

21. Let a container has 4 black balls and 6 white balls. All of them are identical other than the color. Randomly select two balls **without** replacement. Find the probability of getting two balls of the same color.

22. Let $P(X=x)=c(0.6)^x$ for $x=0,\ 1,\ 2,....$, and $P(X=x)=0$ otherwise. Find the value of c.

23. Let $P(X=x)=\left(\frac{1}{2}\right)^x$ for $x=1,2,3,....$, and $P(X=x)=0$ otherwise. Let $A=\{1,\ 5,\ 9,...\}$. Find $P(A)$.

24. If the probability mass function of a random variable X is given by

$f(x)=\left(\frac{1}{2}\right)^x$ for $x=1,\ 2,\ 3,\ \ldots,$ then find the $\frac{P(X\geq 5)}{P(X\geq 2)}$.

25. If the probability mass function of a random variable X is given by

$f(x)=\left(\frac{1}{2}\right)^x$ for $x=\ 1,\ 2,\ 3,\ldots,$ then find the $P(X\geq 5\,|\,X>4)$.

26. If the probability mass function of a random variable X is given by $f(x)=\frac{c}{x(x+1)}$ for $x=1,2,3,\ \ldots,$ then find the value of $P(X=1)$.