Notation and Definitions:

- $\gcd(a, b)$ means the *greatest common divisor* of the positive integers a and b.

- $\operatorname{lcm}(a, b)$ means the *least common multiple* of the positive integers a and b.

- If $\gcd(a, b) = 1$, we say a and b are *relatively prime*.

- $\varphi(n)$ is *Euler's phi function*, which is the number of positive integers less than or equal to n which are relatively prime to n.

- $a \equiv b(\bmod\ m)$ if $a - b$ is an integer multiple of m.

- $\lceil x \rceil$ denotes the "ceiling" of x, the smallest integer greater than or equal to x.

- $\lfloor x \rfloor$ denotes the "floor" of x, the greatest integer less than or equal to x.

- $\binom{a}{b} = \frac{a!}{b!(a-b)!}$ denotes the binomial coefficient.

Instructions: Write your answer on the indicated line of the answer sheet **exactly as directed** in each problem. Only the answer sheet will be graded.

1. List all positive divisors of 258 in **increasing order**.

2. How many positive divisors does $2^4 \cdot 3^7 \cdot 7^3 \cdot 11^2$ have?

3. Please find $\gcd(360, 244)$.

4. Please find $\operatorname{lcm}(36, 40)$.

5. What is the exponent on 2 in the prime factorization of 720?

6. What is the exponent on 5 in the prime factorization of 800?

7. Suppose that a, b are positive integers and we know that $\operatorname{lcm}(a, b) = 5580$ and $a \cdot b = 11160$. Please find $\gcd(a, b)$.

8. Please find the 20th prime.

Questions 9-11 refer to the following definition of a, b:
Suppose that a, b have prime factorizations

$$a = 2^5 \cdot 3^7 \cdot 5^9$$
$$b = 2^3 \cdot 3^8 \cdot 11^2$$

9. What is the exponent on 2 in the prime factorization of $\gcd(a, b)$?

10. What is the exponent on 3 in the prime factorization of $\operatorname{lcm}(a, b)$?

11. What is the exponent on 11 in the prime factorization of $\operatorname{lcm}(a, b)$?

12. What is the smallest possible positive value for c satisfying the equation below, assuming that x, y are arbitrary integers?

$$14x + 21y = c.$$

13. Please find $\gcd(129, 473)$.

14. Which of the numbers below is divisible by 3? **Write either "x", "y", or "none" on the answer sheet.**

$$x = 12,342,343$$
$$y = 342,428$$

15. Which of the numbers below is divisible by 4? **Write either "x", "y", or "none" on the answer sheet.**

$$x = 498,234$$
$$y = 423,424$$

16. Please calculate $\lceil \sqrt{180} \rceil$.

17. Please calculate $\lfloor \sqrt{220} \rfloor$.

18. Please calculate $\varphi(35)$.

19. Please calculate $\varphi(43)$.

20. Please calculate $\varphi(64)$.

21. Please calculate $\binom{18}{4}$.

22. What is the coefficient of $a^4 b$ in the simplified expansion of $(2a - b)^5$?

23. Please calculate:
$$\binom{5}{0} + \binom{5}{1} + \binom{5}{2} + \binom{5}{3} + \binom{5}{4} + \binom{5}{5}.$$

24. Please convert the base 2 (binary) number 1010111_2 to base 10.

25. Please convert the base 10 number 113 to base 5.

26. What values of $x \in \{0, 1, 2, 3, 4, 5, 6\}$ satisfy $x + 4 \equiv 3 \pmod{7}$?

27. What values of $x \in \{0, 1, 2, 3, \ldots 9\}$ satisfy $x^3 \equiv 4 \pmod{10}$?

28. Please calculate $3^{2023} \pmod 5$ (**answer as an integer x, $0 \le x \le 4$**)

29. Please calculate $80^{2023} \pmod 9$ (**answer as an integer x, $0 \le x \le 8$**)

30. Please calculate $131^{2023} \pmod{11}$ (**answer as an integer x, $0 \le x \le 10$**)

Thank you for participating in the Pittsburg State Math Relays!