Marathon–2023

Do not write on the test below but only on the plain paper provided. Answers put on the form below will not be graded.

1. Say you have a deck of 52 cards and you deal them to 5 players, not dealing to yourself, so that everyone has the exact same number as everyone else. Suppose that you deal as many rounds as possible, maintaining that everyone has the same number of cards. Answer the following questions.

   (a) How many cards does each player have?

   (b) How many cards does the you have left over?

2. We deal cards again as in Problem 1. This time we are dealing from a deck of n cards to d players. We call the number of cards each player receives the *quotient* of n divided by d and the number of cards left over as the *remainder* of n divided by d. Find the quotient, q, and remainder, r, for each of the following n and d.

   (a) $n = 16$, $d = 3$

   (b) $n = 72$, $d = 7$

   (c) $n = 360$, $d = 30$

   (d) $n = 640$, $d = 100$

3. Given integers n and d with quotient and remainder as defined in Problem 2, let $n//d$ denote the quotient of n divided by d and $n \% d$ denote the remainder. Calculate the following.

   (a) $1729//10$, $1729 \% 10$

   (b) $1729//100$, $1729 \% 10$

4. Given $n = 900$ and $d = 210$, do the following.

   (a) Factor n into prime factors.

   (b) Factor d into prime factors.

   (c) Find the largest number that will divide both n and d with no remainder.

5. Using the notation from Problem 3, it is a fact that $n = d \cdot (n//d) + (n \% d)$.

   (a) Given $n = 4095$ and $d = 8$, let $n_1 = n//d$ and $r_1 = n \% d$. Find n_1 and r_1.

   (b) When n_i and r_i are given, define $n_{i+1} = n_i//8$ and $r_{i+1} = n_i \% 8$. Find all pairs $(n_1, r_1), (n_2, r_2), \ldots, (n_i, r_i), \ldots$ until $n_i = 0$.

6. Let $f(x) = 7x^3 + 7x^2 + 7x + 7$. Evaluate $f(x)$ for the following values of x.

   (a) $x = 10$

   (b) $x = 8$

   (c) How does this relate to Problem 5?

7. Given numbers n_i and n_{i+1}, define $n_{i+2} = n_i \% n_{i+1}$.

   (a) Let $n_0 = 900$ and $n_1 = 210$, and compute n_i until it is equal to 0.

   (b) What is the last nonzero value?

   (c) How does the answer in 7b compare to the answer to Problem 4c?

8. Render the following base 8 numbers into base 10.

   (a) $(762)_8$

   (b) $(695)_8$

   (c) $(927)_8$

9. Render the following base 10 numbers into base 8.

   (a) 857

   (b) 295

   (c) 1023

   (d) 1922

10. Find the greatest common divisors of the following pairs of numbers.

   (a) $1957, 717$

   (b) $8957, 1952$

   (c) $9992, 2291$