Math Relays 2023

Analytic Geometry

Select the most appropriate letter and shade in the corresponding region on the answer sheet. Choice "none" represents "none of these".

1. What is the point Q if, for $P = (-3, 6)$, the midpoint of PQ happens to be $(2, 2)$?

(a) $(4, 4)$ (b) $(7, -2)$ (c) $(3, 4)$ (d) $(4, 3)$ (e) none

2. What is Q if the distnce between $(1 + 3/5, 2 + 4/5)$ and Q is 1?

(a) $(1, 2)$ (b) $(2, -2)$ (c) $(2, -1)$ (d) $(1, 1)$ (e) none

3. In a triangle whose radius of circumcircle is 7, the angle opposite the side of length 7 is?

(a) 45^0 (b) 60^o (c) not enough info (d) 30^0 (e) none

4. The point $(\pi, 3)$ is on the ellipse

(a) $\frac{x}{\pi} = y$ (b) $\frac{x}{\pi} = 3$ (c) $\frac{x^2}{\pi^2} = 3$ (d) $\frac{x^2}{\pi^2} = 1 - \frac{y^2}{3}$ (e) none

5. The equation $x^2 - y^2 = 25$ describes

(a) a hyperbola (b) a parabola (c) two lines (d) an ellipse (e) none

6. The area of a triangle with sides $3, 8, 7$ and radius of a circle inscribed in the triangle $= 3$, is

(a) 18 (b) 27 (c) 27/2 (d) 36 (e) none

7. For $A > 0$, $B > 0$, and $E < 0$, the equation $Ax^2 + By^2 + Cx + Dy + E = 0$ describes

(a) an ellipse (b) a hyperbola (c) a parabola (d) all of these (e) none

8. The slope of the line is undefined if

(a) $y = \infty$ (b) $y = x$ (c) $x = \infty$ (d) $x = 0$ (e) none

9. An equation of the line that makes a 45^o angle with the line $x = 0$ and passes through $(-2, -1)$ is

(a) $y = -\frac{x}{2} - 2$ (b) $y = x + 1$ (c) $y = 3x + 5$ (d) $y = 2x + 3$ (e) none

10. The slope of the line that intersects the line $7x + 21y = 35$ at $(2, 1)$ and passes through $(5, 0)$ is

(a) -3 (b) $-\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 2 (e) none

11. The parabola $x^2 - 2xy - x + y^2 = 0$ opens

(a) upward (b) downward (c) in the direction of $y = x$ (d) in the direction of $y = -x$ (e) none

12. The equation $(x + y + z)^2 - (x - y - z)^2 = 0$ describes

(a) two planes (b) a sphere (c) a cone (d) a hyperboloid (e) none

13. The acute angle between the asymptotes of the hyperbola $3x^2 - y^2 = 1$ is

(a) 60^o (b) 30^o (c) 45^o (d) 75^o (e) none

14. $(x - 2y)^2 + (x + 2y)^2 = 1$ is an equation of

(a) a circle (b) a parabola (c) a hyperbola (d) an ellipse that isn't a circle (e) none

15. A circle tangent to $x^2 + y^2 - 2y = 0$ is

(a) $x^2 + y^2 + 4y = 4$ (b) $x^2 + (y - 2)^2 = 4$ (c) $(x - 2)^2 + y^2 = 4$ (d) $(x - 2)^2 + (y - 2)^2 = 4$ (e) none

16. The center of the ellipse $(3y + 6)^2 + (4x - 8)^2 = 0$ is

(a) $(2, -3)$ (b) $(\frac{1}{2}, -\frac{1}{3})$ (c) $(-3, 2)$ (d) $(\frac{1}{3}, -\frac{1}{2})$ (e) none

17. The equation $3x + y - 7 = 0$ in (x, y, z) describes

(a) a line (b) a cylinder (c) the empty set (d) a plane (e) none

18. The x-intercept of the line L: $x = 1 - 6t$, $y = -1 + 3t$, where t is a real number, is

(a) $\frac{1}{3}$ (b) 0 (c) -1 (d) 1 (e) none