

Analysis

PSU Math Relays 2023

- For each problem place your answer in the appropriate blank on the answer sheet provided.
- Simplify each answer as far as possible. Write numerical answers in exact form, such as fractions or radicals, rather than decimal approximations.
- You may **not** use a calculator on this test.

In problems 1–4 find the indicated limit.

1. $\lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{x^2 - 1}$
2. $\lim_{x \rightarrow \infty} \frac{x + \ln x}{x}$
3. $\lim_{\theta \rightarrow \pi} \tan(\sin(\theta))$
4. $\lim_{x \rightarrow -1^+} f(x)$, where $f(x) = \begin{cases} 2x^2 - x & \text{if } x \leq -1 \\ x^2 + 1 & \text{if } x > -1 \end{cases}$

In problems 5–8 Let $f(x) = x + \frac{1}{x}$.

5. $f(-1) = ?$
6. $f'(-1) = ?$
7. $f''(-1) = ?$
8. $f^{(3)}(-1) = ?$

In problems 9–12 find the indicated derivative.

9. $y = \sqrt{x^2 - 2x + 3}$, $\frac{dy}{dx} = ?$
10. $f(x) = x^2 e^x$, $f'(x) = ?$
11. $r = \frac{2 + \sin \theta}{2 - \cos \theta}$, $\frac{dr}{d\theta} = ?$
12. $f(x) = \int_1^{-x} \sqrt{5 + t^2} dt$, $f'(x) = ?$

In problems 13–16 let $f(x) = x^3 - 3x^2 - 9x + 27$. Use the interval notation (a, b) to write intervals in your answers.

13. Find the interval(s) on which f is increasing.
14. Find the interval(s) on which f is decreasing.
15. Find the interval(s) on which f is concave up.
16. Find the interval(s) on which f is concave down.

In problems 17–19, determine whether the statement is *true* or *false*. The value c always refers to an interior point of the domain of function $f(x)$.

17. If $f''(c) > 0$, then $f(c)$ is a local minimum.
18. If $f'(c) = 0$, then $f(x)$ has a local extreme value at $x = c$.
19. If $f(x)$ is differentiable and it has a local extreme value at c , then $f'(c) = 0$.

In problems 20–23 evaluate the indicated integral.

20. $\int_0^4 (2x + 1)^{2/3} dx$

21. $\int_0^2 \frac{2x dx}{x^2 + 2}$

22. $\int te^{-t^2} dt$

23. $\int_0^{\pi/2} \sin(3x) dx$

24. Find the slope-intercept form of the equation for the tangent line to the curve $y = 4 - x^2$ at $x = -1$.
25. Use the definition of average value in terms of area to find the average value of the function $f(x) = \sin^2 x$ on the interval $[0, \pi]$.
26. Find the area of the finite region bounded by the curve $y = x^2 - 2x - 8$ and the line $y = 2x - 3$.