NO CALCULATORS

ALGEBRAIC EQUATIONS & INEQUALITIES

Pitt State Math Relays 2023

For each question below, completely shade in the shape next to the letter of the **simplest** correct answer in the appropriate space on the answer sheet. Only the answer sheet will be graded. Unless stated otherwise, solve for *x* in the following.

1. $2x-5=-5-2x$

   (A) $\frac{-5}{2}$ (B) 0 (C) $\frac{4}{5}$ (D) all real numbers (E) none of these

2. $2x+9 \le 13$

   (A) $x \le 3$ (B) $-2 \le x \le 0$ (C) $-2 \le x \le 3$ (D) $x \le 2$ (E) none of these

3. $12x^2+14x=6$

   (A) $\frac{3 \pm \sqrt[4]{7}}{2}$ (B) 0,1 (C) $\frac{-5}{2}, \frac{7}{3}$ (D) $\frac{1}{3}, \frac{-3}{2}$ (E) none of these

4. One of the solutions of $x^2+20=-21x$ is

   (A) $3i$ (B) - 20 (C) $-7+4i$ (D) 2 (E) none of these

5. $5\left(-5x+5+5(x+5)\right)=5x+5$

   (A) $-\frac{7}{3}$ (B) -5 (C) $\frac{17}{2}$ (D) 29 (E) none of these

6. If for all values of *x*, $\left(x-c\right)^2=c^2+x^2$, then *c* =

   (A) 0 (B) 2 or $-\frac{1}{2}$ (C) 1 (D) -2 (E) none of these

7. $\log_{10} x=-4$

   (A) $\frac{5}{2}$ (B) 10,000 (C) $\frac{1}{10,000}$ (D) 100,000 (E) none of these

8. $81^{1+x}=27^{2x-3}$

   (A) 4 (B) $\frac{13}{2}$ (C) $\frac{1}{3}$ (D) -1 (E) none of these

9. In the equation $x^2 - 7x + 2 = 0$, the sum of the roots is

(A) 9 (B) 7 (C) -7 (D) -9 (E) none of these

10. In the equation $x^2 - bx + c = 0$, the product of the roots is

(A) b (B) $-b$ (C) c (D) $b^2 - 4c$ (E) none of these

11. If $f(x) = x^2 - cx - 2$ and $f(2) = 10$, then $c =$

(A) -4 (B) 4 (C) -5 (D) 8 (E) none of these

12. Find the value of y in the solution. $\begin{cases} 4x - 3y = 25 \\ 5x - 12y = 12 \end{cases}$

(A) $\frac{7}{3}$ (B) 8 (C) -9 (D) 9 (E) none of these

13. $7^x = 19$

(A) no solution (B) $\frac{19}{7}$ (C) $\log_{19}\left(\frac{19}{7}\right)$ (D) $\log_7 19$ (E) none of these

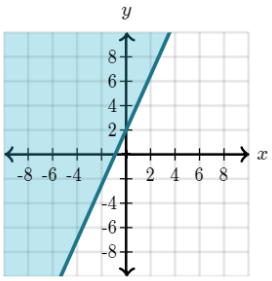
14. For which equation does the product of the roots equal $\frac{3}{4}$ and the sum of the roots equal -2?

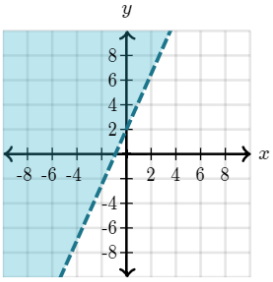
(A) $4x^2 - 8x + 3 = 0$ (B) $4x^2 + 8x + 3 = 0$ (C) $4x^2 - 3x - 8 = 0$ (D) $4x^2 + 3x - 2 = 0$
(E) none of these

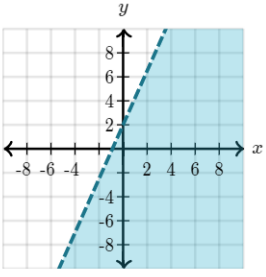
15. The graph of the equation defined by $3x^2 - 5 = -y^2$ is a

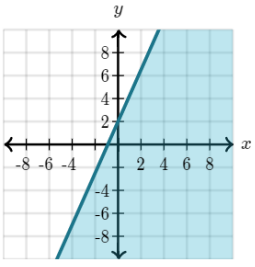
(A) circle (B) ellipse (C) hyperbola (D) parabola (E) none of these

16. Select the graph which best represents $8 + 9x - 4y < 0$.

(A) 

(B) 

(C) 

(D) 

(E) None of these

17. $x^2 - 3x - 1 \le 17$

(A) $-6 \le x \le 3$ (B) $x \le -6$ or $x \ge 3$ (C) $x \le -3$ or $x \ge 6$ (D) $-3 \le x \le 6$

(E) none of these

18. $\dfrac{18}{x-6} + 2 = \dfrac{6}{x-6}$

(A) 0 (B) $2, -\dfrac{1}{2}$ (C) 6 (D) -18 (E) none of these

19. $2\log_3(5x) = 4$

(A) 6.4 (B) 2.56 (C) $\dfrac{9}{5}$ (D) $\dfrac{8}{5}$ (E) none of these

20. $|x+2|+6 \le 11$

(A) $x < 3$ (B) $-7 \le x \le 0$ (C) $-7 \le x \le 3$ (D) all real numbers (E) none of these

21. Find the value of x in the solution. $\begin{cases} 2x = -y + 14 \\ 5x = 4y + 9 \end{cases}$

(A) 8 (B) 5 (C) 4 (D) 9 (E) none of these

22. What is the solution set of the equation $|x^2 - 2x| = 8x - 16$?

(A) $\{\pm 8\}$ (B) $\{2\}$ (C) $\{2, 8\}$ (D) $\{2, \pm 3\}$ (E) none of these

23. The set of all points in the plane 5 units from the x-axis is given by the equation

(A) $x^2 + y^2 = 25$ (B) $xy = -5$ (C) $y = 5$ (D) $|y| = 5$ (E) none of these

24. A starship fires a laser which travels along the straight line given by the equations $\begin{cases} x = 3t - 1 \\ y = t - 5 \\ z = 2t + 4 \end{cases}$ where t is the number of seconds since it was fired. What point has the laser reached at time $t = 2$ seconds?

(A) $(3,1,2)$ (B) $(-1,-5,4)$ (C) $(5,-3,8)$ (D) $(8,-2,10)$ (E) none of these

25. $\dfrac{1}{x-1} + \dfrac{2x}{x+2} = \dfrac{5x^2}{x^2 + x - 2}$

(A) 0 (B) $-1, \dfrac{2}{3}$ (C) $1, -2$ (D) -15 (E) none of these

26. $4(x-5)^2-64=0$

(A) $1, 9$ (B) $-1, -9$ (C) $4\pm\sqrt{5}$ (D) $-4\pm\sqrt{5}$ (E) none of these

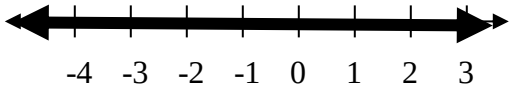
27. $|x-3|+3\le 1$

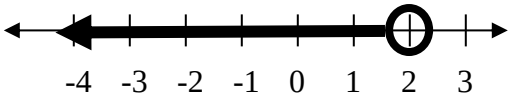
(A) $x<3$ (B) $-7\le x\le 0$ (C) $-7\le x\le 3$ (D) no solution (E) none of these

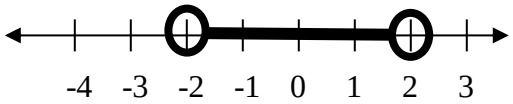
28. $\sqrt{x+6}+\sqrt{2-x}=4$

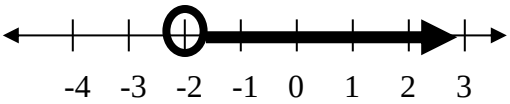
(A) $\sqrt{31}, -2$ (B) -2 (C) $2, -2$ (D) no solution (E) none of these

29. The graph of the solution set for $20-7x>6$ is

(A) 

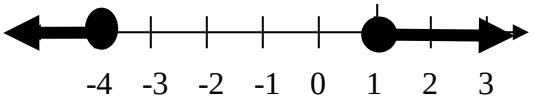
(B) 

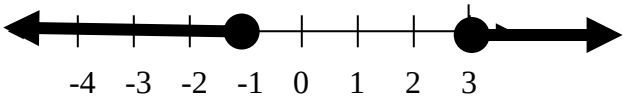
(C) 

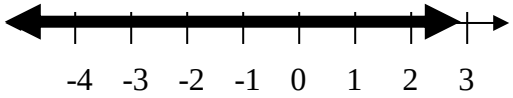
(D) 

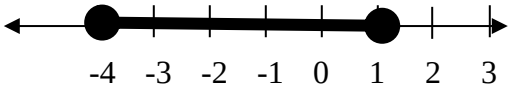
(E) none of these

30. The graph of the solution set for $x^2+3x\ge 4$ is

(A) 

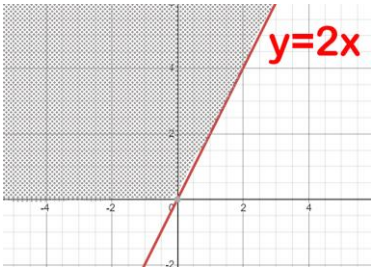
(B) 

(C) 

(D) 

(E) none of these

31. Which of the inequalities best describes the shaded region in the figure.



(A) $y\ge 2x$ and $y\le 0$ (B) $y\ge 2x$ and $y\ge 0$ (C) $y\le 2x$ and $y\ge 0$

(D) $y\le 2x$ and $y\le 0$ (E) none of these

32. The equality $4^3=64$ can be written as

(A) $\log_{64} 3=4$ (B) $\log_3 64=4$ (C) $\log_4 64=3$ (D) $\log_3 4=64$ (E) none of these

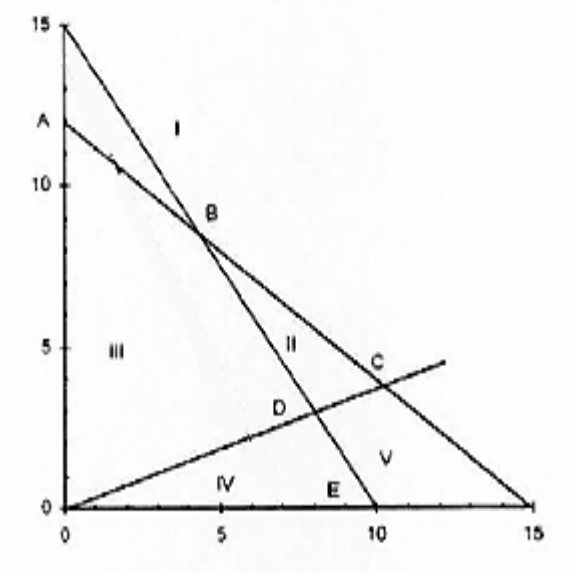
33. An equation of the line through $(2,0)$ and $(-3,5)$ is

(A) $y=-x+2$ (B) $y=3x-8$ (C) $y=3x+2$ (D) $y=x-4$ (E) none of these

34. The equation $x^2+1=2x$ has

(A) two distinct real solutions (B) one real solution (C) one real solution and one imaginary solution (D) no real solutions (E) none of these

35. The solution set of $\begin{array}{l} 3x-8y\ge 0 \\ 12x+15y\le 180 \\ 3x+2y\ge 30 \\ x,y\ge 0 \end{array}$ is which region?



(A) I (B) II (C) III (D) IV (E) V

36. $(x+3)^2-(x+2)^2=x+2$

(A) -3 (B) 3 (C) $\pm\frac{1}{3}$ (D) $\frac{1}{3}$ (E) none of these

37. $x^{-2}-1=15$

(A) 4 (B) -4 (C) $\pm\frac{1}{4}$ (D) $\frac{1}{4}$ (E) none of these

38. $\frac{x^3+1}{x+1}-(x-1)^2=3$

(A) 3 (B) 3 or $-\frac{1}{2}$ (C) $-4, 3$ or $\frac{1}{2}$ (D) -4 or $\frac{1}{2}$ (E) none of these

39. Find all roots of $x^4-6x^2+8=0$.

(A) 2, 4 (B) $\pm 2\pm\sqrt{2}$ (C) $\pm\sqrt{2},\ \pm 2$ (D) $\sqrt{2}+2i, \sqrt{2}-2i, 2+i\sqrt{2}, 2-i\sqrt{2}$ (E) none of these

40. Solve the inequality $\frac{-x^2-x+6}{x^2+2x-3}\ge 0$.

(A) $x\le -3$ or $-2<x<-1$ or $x>1$ (B) $x<-3$ or $x\ge 1$ (C) $1<x\le 2$

(D) $x<1$ or $x\ge 2$ (E) none of these