

Analysis

PSU Math Relays 2019

- For each problem place your answer in the appropriate blank on the answer sheet provided.
- Simplify each answer as far as possible. Write numerical answers in exact form, such as fractions or radicals, rather than decimal approximations.
- You may **not** use a calculator on this test.

In problems 1–4 find the indicated limit.

1. $\lim_{x \rightarrow 1} \sqrt[3]{\frac{x-1}{x+1}}$
2. $\lim_{x \rightarrow \infty} \frac{3x^2 + 7x - 10}{-x^2 + 5x}$
3. $\lim_{x \rightarrow \infty} \cos\left(\frac{x + \sin x}{x^2}\right)$
4. $\lim_{x \rightarrow 0^-} f(x)$, where $f(x) = \begin{cases} x^2 - 2 & \text{if } x \leq 0 \\ 2x - 3 & \text{if } x > 0 \end{cases}$

In problems 5–7 Let $f(x) = \cos(2x)$.

5. $f'(\pi/6) = ?$
6. $f''(\pi/6) = ?$
7. $f^{(5)}(\pi/6) = ?$

In problems 8–11 find the indicated derivative.

8. $g(\theta) = \sec \theta + \cot \theta$, $g'(\theta) = ?$
9. $f(x) = \frac{x^2 - 1}{x^2 + 1}$, $f'(x) = ?$
10. $y = \ln(\sqrt{x^2 + 2})$, $\frac{dy}{dx} = ?$
11. $f(t) = \int_{\sin t}^2 \sqrt{1 - x^2} \, dx$, $f'(\pi/4) = ?$

12. Find the slope-intercept form of the equation for the tangent line to the curve defined by the equation $x^3 + y^4 = 2$, at the point $(x, y) = (1, -1)$.

In problems 13–16 let $f(x) = x + \frac{1}{x}$. Use the interval notation (a, b) to write intervals in your answers.

13. Find the interval(s) on which f is increasing.
14. Find the interval(s) on which f is decreasing.
15. Find the interval(s) on which f is concave up.
16. Find the interval(s) on which f is concave down.

In problems 17–18 let $f(x)$ be a function with derivative $f'(x) = x^2(x^2 - 4x + 3)$.

17. Find the interval(s) on which f is increasing.
18. Find the interval(s) on which f is decreasing.

19. Evaluate the sum $\sum_{n=1}^{40} (n^2 - 3n + 5)$.

In problems 20–23 evaluate the indicated integral.

20. $\int \frac{2x^3 - x^2 + 4}{x^2} dx$

21. $\int_{-1}^3 \frac{x - 1}{x^2 - 2x - 4} dx.$

22. $\int_1^e \frac{\ln x}{x} dx$

23. $\int \sin^2 x dx$

24. Use the definition of average value in terms of area to find the average value of the function $f(x) = \sqrt{16 - x^2}$ on the interval $[-4, 4]$.
25. Find the area of the region enclosed by the parabola $y = 2x - x^2$ and the line $y = x - 2$.