PSU MATH RELAYS LOGIC & SET THEORY 2017

MULTIPLE CHOICE. There are 40 questions. Select the letter of the most appropriate answer and SHADE in the corresponding region of the answer sheet. If the correct answer is NOT one of the choices, mark "E" on the answer sheet. Remember, NO CREDIT is given for any marks on the answer sheet except the complete shading of the correct choice.

Write a negation for the statement.
1) 12x + 11y < 12
   A) 12 > 12x + 11y  B) 12x + 11y ≥ 12  C) 12x + 11y ≤ 12  D) 12x + 11y > 11

Translate the symbolic compound statement into words.
2) Let p represent the statement: "Students are happy" and let q represent the statement: "Teachers are happy." ~ (p ∨ ~ q)
   A) It is not the case that students are happy or teachers are not happy.
   B) Students are not happy and teachers are not happy.
   C) It is not the case that students are happy and teachers are not happy.
   D) Students are not happy or teachers are not happy.

Write the negation of the statement. If not a statement, state so.
3) The following excerpt appears on the cover page of a mathematics exam: We will deduct points if there is no work to justify your answer.
   A) We may not deduct points if there is no work to justify your answer.
   B) We will not deduct points if there is no work to justify your answer.
   C) We will not deduct points if there is work to justify your answer.
   D) The excerpt is not a statement.

4) The following excerpt appears on the cover page of a mathematics exam: Make sure that you fill in the circle on the answer sheet that corresponds to your answer choice.
   A) Do not make sure that you fill in the circle on the answer sheet that corresponds to your answer choice.
   B) Do not make sure that you do not fill in the circle on the answer sheet that corresponds to your answer choice.
   C) Make sure that you do not fill in the circle on the answer sheet that corresponds to your answer choice.
   D) The excerpt is not a statement.

Give the number of rows in the truth table for the compound statement.
5) (p ∨ q) ∧ (~ r ∨ s) ∨ t
   A) 25  B) 10  C) 8  D) 32

Find the number of subsets of the set.
6) {math, English, history, science, art}
   A) 32  B) 24  C) 28  D) 16

7) {x | x is an even number between 17 and 37}
   A) 36  B) 8  C) 256  D) 1024

Please turn over for additional questions
Construct a truth table for the compound statement.

8) \( \neg s \land \neg p \)

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9) \( \neg [\neg (q \lor s)] \)

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Construct a truth table for the statement.

10) \( (p \rightarrow q) \rightarrow (\neg p \lor q) \)

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11) \( \sim(p \rightarrow q) \rightarrow (p \land \sim q) \)

<table>
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Write the statement as an equivalent statement that does not use the if . . . then connective. Remember that \( p \rightarrow q \) is equivalent to \( \sim p \lor q \).

12) If the sun comes out tomorrow, the roses will open.
   A) The sun does not come out tomorrow and the roses will not open.
   B) The sun does not come out tomorrow or the roses will not open.
   C) The sun does not come out tomorrow or the roses will open.
   D) The sun comes out tomorrow and the roses will not open.

Write the negation of the statement. Remember that the negation of \( p \rightarrow q \) is \( p \land \sim q \).

13) If it is raining, you take your umbrella.
   A) If it is raining, you do not take your umbrella.
   B) It is raining and you do not take your umbrella.
   C) It is not raining and you do not take your umbrella.
   D) It is not raining and you take your umbrella.

14) If \( 7x + 2y > 3 \), the answer is "Lake".
   A) \( 7x + 2y > -3 \), so the answer is not "Lake".
   B) \( 7x + 2y \leq 3 \) and the answer is not "Lake".
   C) If \( 7x + 2y > 3 \), the answer is not "Lake".
   D) \( 7x + 2y > 3 \) and the answer is not "Lake".

Let \( U = \{q, r, s, t, u, v, w, x, y, z\} \); \( A = \{q, s, u, w, y\} \); \( B = \{q, s, y, z\} \); and \( C = \{v, w, x, y, z\} \). List the members of the indicated set, using set braces.

15) \((A \cap B)'
   A) \{t, v, x\}
   B) \{q, s, t, u, v, w, x, y\}
   C) \{t, u, v, w, x, z\}
   D) \{s, u, w\}

16) \(B' \cap (A \cup C)'
   A) \{t, u\}
   B) \{q, r, s, t, u, w\}
   C) \{q, r, s, t, u, v, w, x, y\}
   D) \{t, u, w\}

Use the union rule to answer the question.

17) If \( n(A) = 4 \), \( n(B) = 9 \), and \( n(A \cap B) = 2 \); what is \( n(A \cup B) \)?
   A) 11
   B) 12
   C) 13
   D) 10
Shade the Venn diagram to represent the set.

18) $A' \cap B'$

19) $(A \cap B) \cup (A \cup B)'$
Use a Venn Diagram and the given information to determine the number of elements in the indicated region.

20) $n(U) = 60$, $n(A) = 34$, $n(B) = 22$, and $n(A \cap B) = 8$. Find $n(A \cup B)'$.
   
   A) 48  B) 4  C) 56  D) 12

21) $n(A \cup B \cup C) = 77$, $n(A \cap B \cap C) = 11$, $n(A \cap B) = 24$, $n(A \cap C) = 21$, $n(B \cap C) = 19$, $n(A) = 56$, $n(B) = 38$, and $n(C) = 36$. Find $n(A' \cap B \cap C)$.
   
   A) 7  B) 9  C) 10  D) 8

Decide whether the argument is valid or invalid, and give the form (of valid or invalid argument) that applies.

22) If I'm hungry, then I will eat.
   
   I'm not hungry.
   
   A) Valid; modus ponens  B) Invalid; fallacy of the inverse
   
   C) Valid; modus tollens  D) Invalid; fallacy of the converse

Use a Venn diagram to answer the question.

23) A survey of 240 families showed that
   
   91 had a dog;
   70 had a cat;
   31 had a dog and a cat;
   91 had neither a cat nor a dog, and in addition did not have a parakeet;
   7 had a cat, a dog, and a parakeet.
   
   How many had a parakeet only?
   
   A) 19  B) 24  C) 34  D) 29

NOTE the following section is BI-MODAL for answer choices.

Determine if the argument is valid.

24) If a number is even, then it is divisible by 2.
   
   If a number is divisible by 8, then it is divisible by 2.
   
   Some numbers are either even or divisible by 8.

   A) Invalid  B) Valid

The argument has a true conclusion. Identify the argument as valid or invalid.

25) Rational numbers are real numbers.
   
   Integers are rational numbers.

   A) Valid  B) Invalid

Decide whether the statement is true or false.

26) $\{10, 11, 2\} \cap \emptyset = \{10, 11, 2\}$

   A) True  B) False

Let $p$ represent a true statement, and let $q$ and $r$ represent false statements. Find the truth value of the given compound statement.

27) $\neg(p \wedge \neg q) \lor (\neg r \lor \neg p)$

   A) True  B) False
Use a truth table to decide if the statements are equivalent.
28) \( \neg(q \rightarrow p); q \land \neg p \)
   A) Not equivalent       B) Equivalent

The argument has a true conclusion. Identify the argument as valid or invalid.
29) \( \sqrt{12} \) is less than 12.
    6 is less than 12.
    \( \sqrt{12} \) is less than 6.
       A) Invalid       B) Valid

Use a truth table to decide if the statements are equivalent.
30) \( q \land \neg p; \neg p \rightarrow \neg q \)
    A) Equivalent       B) Not equivalent

Tell whether the statement is true or false.
31) \( 11 \notin \{x \mid x \text{ is an even counting number} \} \)
    A) True       B) False

32) \( \{x \mid x \text{ is a counting number greater than 32} \} = \{32, 33, 34, \ldots \} \)
    A) True       B) False

Insert "\( \subseteq \)" or "\( \not\subseteq \)" in the blank to make the statement true.
33) \( \emptyset \; \; \; \emptyset \)
    A) \( \subseteq \)       B) \( \not\subseteq \)

34) \( \{x \mid x \text{ is a counting number larger than 5}\} \; \; \; \{7, 8, 9, \ldots \} \)
    A) \( \subseteq \)       B) \( \not\subseteq \)

Let \( A = \{1, 3, 5, 7\}; \; B = \{5, 6, 7, 8\}; \; C = \{5, 8\}; \; D = \{2, 5, 8\}; \) and \( U = \{1, 2, 3, 4, 5, 6, 7, 8\} \). Determine whether the given statement is true or false.
35) \( \emptyset \subseteq A \)
    A) True       B) False

36) \( U \subseteq A \)
    A) True       B) False

37) \( \{0\} \subseteq U \)
    A) True       B) False

38) \( C \not\subseteq B \)
    A) True       B) False

39) \( A \subset A \)
    A) True       B) False

Let \( p \) represent a true statement, and let \( q \) and \( r \) represent false statements. Find the truth value of the given compound statement.
40) \( \neg(p \land \neg q) \lor \neg q \)
    A) True       B) False

Thank you for participating!